Black hole giants

Aninda Sinha and Julian Sonner

Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, U.K.
E-mail: `a.sinha@damtp.cam.ac.uk`, `j.sonner@damtp.cam.ac.uk`

ABSTRACT: We investigate giant and dual giant type BPS configurations in the near-horizon geometry of a certain $\frac{1}{16}$-BPS AdS$_5$ black hole. By quantising the space of solutions we count the dual giant configurations and compare with the black hole entropy. This suggests a missing degeneracy factor which we argue comes from an angular momentum quantum number. From the D-brane world volume this arises from BPS electromagnetic waves. We study these waves in the context of giants and dual giants in the black hole near-horizon geometry. We further demonstrate that turning on waves on the world-volume of $\frac{1}{8}$-BPS dual giants in AdS$_5 \times$ S^5 leads to $\frac{1}{16}$-BPS states with an additional angular momentum quantum number.

Contents

1. Introduction

String theory has been successful in accounting for the statistical entropy of many supersymmetric asymptotically flat black holes [1, 2]. Three years ago Gutowski and Reall discovered supersymmetric asymptotically AdS$_5$ black holes with regular horizons [3−8]. A microscopic understanding of these black holes is an important open problem in AdS/CFT.

The simplest such asymptotically AdS$_5$ black hole rotates with equal angular momenta in two orthogonal planes in AdS$_5$ directions and carries a single U(1) electric charge. The entropy of this black hole is known to be

$$S_{\text{BH}} = \frac{\pi^2}{2G_5}\omega^3\sqrt{1 + \frac{3\omega^2}{4l^2}}\,, \tag{1.1}$$

where ω is a parameter related to the black hole angular momentum and electric charge and l is the AdS$_5$ radius. As was shown in [11], when lifted to a 10-dimensional type IIB solution,

---

[1]We thank N. Suryanarayana for collaboration in this section

the geometry asymptotes to $\text{AdS}_5 \times \text{S}^5$ and preserves just two supersymmetries. Since only the five-form flux is turned on, the microstates of this black hole may be thought to be some configuration of multiple giant gravitons [9, 10], which preserve $\frac{1}{16}$ of the supersymmetries of $\text{AdS}_5 \times \text{S}^5$. The construction and counting of such states is proving to be a difficult, and as yet unsolved, problem (for some related progress see $[12-17]$). So it is natural to look for other avenues to address the problem of microstate counting for these black holes. For instance, a Fermi surface model was proposed for a microscopic description for these black holes in [18] where a qualitative agreement was found.

Around two years ago, Strominger and collaborators provided a specific example of a four-charge black hole carrying D0 and D4 charges with near-horizon geometry $\text{AdS}_2 \times \text{S}^2 \times \text{CY}_3$ where near-horizon microstates could account for the entropy [19, 20]. The microstates involved in this derivation did not preserve any of the asymptotic supersymmetries. One reason for this somewhat surprising feature is that supersymmetric quantum mechanics tells us that the microstates preserving the asymptotic supersymmetries are non-normalisable [21] and hence should not be included in the counting. The way out of this conundrum was to transform to global time [22] and use eigenstates of the global Hamiltonian to do the counting. In Poincaré time, these states corresponded to D0 brane states popping in and out of the horizon.

Motivated by this picture the near-horizon geometry of the simplest Gutowski-Reall black hole was studied in some detail in [23]. There it was shown that there is a doubling of supersymmetries near the horizon. The superisometry group of the horizon was found to be $\text{SU}(1, 1|1)$. When lifted to ten dimensions, the near-horizon geometry has a deformed three-sphere $\tilde{\text{S}}^3$ and a deformed five-sphere $\tilde{\text{S}}^5$ with a fibration of the time coordinate of AdS_2 over them. The AdS_2 part of the geometry can be written in both global and Poincare coordinates. We will call D3-branes wrapping three of the $\tilde{\text{S}}^5$ directions, black hole giant gravitons (BHG) while D3-branes wrapping the $\tilde{\text{S}}^3$ will be called black hole dual giants $(\widehat{\text{BHG}})$. It was shown in [23] that giant and dual giant type probes which preserve half the near-horizon supersymmetries exist in the lifted geometry. In AdS_2 Poincaré coordinates, the probes have zero energy and preserve exactly the asymptotic supersymmetries. In AdS_2 global coordinates, the probes have non-trivial Hamiltonians and preserve none of the asymptotic supersymmetries. In this case both BHG and $\widehat{\text{BHG}}$ preserve the same fraction of the near-horizon supersymmetries. One naturally wonders if these near-horizon microstates could be used to account for the microscopic entropy of the black hole. Another reason to expect this to be the case is that the conserved charges of the black hole can be extracted completely from the near-horizon geometry as was shown in [24].

In this paper we quantise the phase space of solutions of the $\widehat{\text{BHG}}$s in AdS_2 global coordinates and count them. We find that there is an exponential degeneracy and hence a large contribution to the microstates from these solutions. The leading order result is off by a degeneracy factor which we argue is the result of a missing quantum number.

Motivated by the missing quantum number we study world-volume fluxes which preserve the same supersymmetry as the original solutions. We find that a whole class of solutions exist where electromagnetic waves can be turned on in the fibre direction after writing the deformed 3-spheres as Hopf fibrations over $\mathbb{C}P^1$. These waves contribute to

the missing angular momentum quantum number. The resulting equations of motion are very similar to the $\frac{1}{8}$-BPS AdS$_5\times$ S^5 giants with fluxes which were studied in [25]. We will demonstrate that turning on world-volume fluxes on $\frac{1}{8}$-BPS AdS$_5\times$S^5 dual giants will generically break supersymmetry by a further half. This can be anticipated by noting that the most general $\frac{1}{8}$-BPS dual giant configuration [26] is known to be spherically symmetric and turning on waves will generically break this spherical symmetry. We provide a simple maximisation argument motivated by [14] to show how the near-horizon and asymptotic states could be used to account for the macroscopic entropy. The direct way of doing this is by quantising the new phase space which we have not attempted in this paper.

In order to account for the full black hole entropy, one possibly needs to turn on mechanical waves on the world-volume as well. We will not have anything to say about these but will leave this as an open problem. In the final solution to this problem from near-horizon microstates, we feel our BPS analysis of world-volume electromagnetic waves will be important. Our analysis may also be helpful in developing an understanding of how the black hole superconformal quantum mechanics is embedded in $\mathcal{N} = 4$ super Yang Mills.

The paper is organised as follows. In section 2, we review the near horizon geometry and probes of the black hole under investigation. In section 3, we count dual giant type configurations and motivate the addition of fluxes on the world-volume. In section 4, we study near horizon giant and dual giant type configurations with world-volume fluxes which preserve the same supersymmetry as those without fluxes. We conclude with a discussion and some speculative comments in section 5. Calculational details of the supersymmetry analysis are given in appendices A and B.

2. Review of the near horizon

2.1 Geometry

The near-horizon-geometry can be written as [23]

$$ds_{10}^2 = ds_5^2 + l^2 \sum_{i=1}^{3} \left[(d\mu_i)^2 + \mu_i^2 \left(d\xi_i + \frac{2}{l\sqrt{3}} A \right)^2 \right], \tag{2.1}$$

$$F^{(5)} = (1 + *_{(10)}) \left[-\frac{4}{l} \text{vol}_5 + \frac{l^2}{\sqrt{3}} \sum_{i=1}^{3} d(\mu_i)^2 \wedge d\xi_i \wedge *_{(5)} F^{(2)} \right], \tag{2.2}$$

where $\mu_1 = \sin\alpha$, $\mu_2 = \cos\alpha\sin\beta$ and $\mu_3 = \cos\alpha\cos\beta$ with $0 \leq \alpha, \beta \leq \pi/2$, $0 \leq \xi_i \leq 2\pi$ and together they parametrise an S^5. Here in Poincaré coordinates for the AdS$_2$ part

$$ds_5^2 = -a^2 r^2 dt^2 + b^2 \frac{dr^2}{r^2} + \frac{\omega^2}{4} \left((\sigma_1^L)^2 + (\sigma_2^L)^2 \right) + \frac{\omega^2}{4a^2b^2} \left(\sigma_3^L + \frac{6a^2b^2}{l\omega} r dt \right)^2, \tag{2.3}$$

where $a^2 = \frac{4\lambda^2}{\omega^2 l^2 \left(1 + \frac{3\omega^2}{4l^2}\right)}$, $b^2 = \frac{\omega^2 l^2}{4\lambda^2}$ and $\lambda = \sqrt{l^2 + 3\omega^2}$. The gauge potential is given by

$$A = \frac{\sqrt{3}}{2} \left(\frac{2r}{\omega} dt + \frac{\omega^2}{4l} \sigma_3^L \right). \tag{2.4}$$

The right-invariant one-forms on SU(2) are

$$\sigma_1^L = \sin\phi d\theta - \sin\theta\cos\phi d\psi\,, \tag{2.5}$$

$$\sigma_2^L = \cos\phi d\theta + \sin\theta\sin\phi d\psi\,, \tag{2.6}$$

$$\sigma_3^L = d\phi + \cos\theta d\psi\,. \tag{2.7}$$

The range of the angles are $0 \le \theta \le \pi$, $0 \le \psi \le 2\pi$ and $0 \le \phi \le 4\pi$. The 10-d Killing spinor is given by

$$\epsilon = \exp\left[-\frac{i}{2}(\xi_1 + \xi_2 + \xi_3)\right]\exp\left[-\frac{2i\lambda rt}{l\omega^2}\Gamma_{49}(1+\Gamma_{09})\right]\times$$
$$\times \exp\left[\left(\frac{3\omega}{4\lambda}\Gamma_{49}(1+\Gamma_{09}) - \frac{1}{2}\Gamma_{09}\right)\ln r\right]\epsilon_0\,, \tag{2.8}$$

where ϵ_0 is a 32 component constant spinor satisfying $\Gamma_{11}\epsilon_0 = -\epsilon_0, \Gamma_{0149}\epsilon_0 = -i\epsilon_0, \Gamma_{23}\epsilon_0 = -i\epsilon_0, \Gamma_{57}\epsilon_0 = -i\epsilon_0$. In terms of global coordinates for the AdS$_2$ part

$$ds_5^2 = -\left(1 + \frac{\rho^2}{b^2}\right)d\tau^2 + \frac{d\rho^2}{1 + \frac{\rho^2}{b^2}} + \frac{\omega^2}{4}\left((\sigma_1^L)^2 + (\sigma_2^L)^2\right) + \frac{\omega^2}{4a^2b^2}\left(\sigma_3^L - \frac{6ab}{\omega l}\rho d\tau\right)^2\,, \tag{2.9}$$

and

$$ds_{S^5}^2 = l^2\left(d\alpha^2 + \cos^2\alpha d\beta^2 + \sum_i \mu_i^2\left(d\xi_i - \frac{\omega^2}{4l^2}\sigma_3 + \frac{2}{\omega lab}\rho d\tau\right)^2\right)\,, \tag{2.10}$$

with

$$A = -\frac{\sqrt{3}}{2}\left(\frac{\omega^2}{4l}\sigma_3^L - \frac{2}{\omega ab}\rho d\tau\right)\,. \tag{2.11}$$

The global coordinate ϕ and Poincare ϕ are related by a ρ, τ dependent transformation which leaves the period invariant. In both coordinate systems, the geometry is that of U(1) fibre bundle with coordinate ϕ over a two-dimensional base sphere with coordinates θ, ψ. The Killing spinor is given by

$$\epsilon = \exp\left[-\frac{i}{2}(\xi_1 + \xi_2 + \xi_3)\right]\exp\left[-\frac{1}{2}\sinh^{-1}\frac{\rho}{b}M\right]\exp\left[-\frac{i}{2}M\Gamma_{49}\frac{\tau}{b}\right]\epsilon_0\,, \tag{2.12}$$

where $M = \frac{2b}{l}(\frac{3}{2}\Gamma_{04} + \frac{l}{\omega ab}\Gamma_{09})$, $M^2 = 1$ and $\Gamma_{11}\epsilon_0 = -\epsilon_0$, $\Gamma_{0149}\epsilon_0 = i\epsilon_0$, $\Gamma_{23}\epsilon_0 = i\epsilon_0$ and $\Gamma_{57}\epsilon_0 = -i\epsilon_0$.

In both coordinate systems there are four independent supersymmetries that the geometry preserves which is twice the number that the full black hole sees.

2.2 Near-horizon probes

In [23], we investigated D3-brane probes without world-volume fluxes in the near horizon geometry. In the conventions of [23] there exist giant-like anti-branes and dual giant-like branes in Poincaré coordinates, which preserve orthogonal supersymmetries. In global coordinates there exist BHG and $\widehat{\text{BHG}}$ solutions preserving the *same* supersymmetries. Let us denote the world-volume coordinates by $\sigma_0, \sigma_1, \sigma_2, \sigma_3$. The BHGs have $\sigma_1 = \beta$, $\sigma_2 = \xi_2$ and $\sigma_3 = \xi_3$ while $\widehat{\text{BHG}}$s have $\sigma_1 = \theta$, $\sigma_2 = \phi$, $\sigma_3 = \psi$. In what follows, we only review brane solutions for the case of global coordinates, rather than anti-brane solutions.

Poincaré BHG and $\widehat{\text{BHG}}$. The Poincaré BHGs and $\widehat{\text{BHG}}$s have $\sigma_0 = t$. All the embedding coordinates are constant and hence $H = 0$. They preserve the supersymmetries obeying $\Gamma_{09}\epsilon_0 = \epsilon_0$ and hence are $\frac{1}{2}$-BPS with respect to the enhanced near-horizon supersymmetries. In both cases the preserved supersymmetry is the same as that of the full black hole. After integrating over the world-volume spatial coordinates, the expression for the non-zero conjugate momenta for BHG are

$$P_\phi = T_3 2\pi^2 l^2 \omega^2 \cos^2\alpha \,, \quad P_\psi = P_\phi \cos\theta \,, \quad P_{\xi_1} = T_3 2\pi^2 l^4 \cos^2\alpha \,. \tag{2.13}$$

Here T_3 is the D3-brane tension which we dropped in our earlier paper.

For the $\widehat{\text{BHG}}$ states the non-zero momenta are

$$P_{\xi_i} = T_3 \pi^2 (\omega^2 l^2 + (2l^2 + \omega^2)\omega^2)\mu_i^2 \,, \tag{2.14}$$

The second term on the r.h.s. arises from the WZ term and can be gauged away. Another way of seeing this is to introduce a fictitious parameter in front of it and note that the solutions are invariant under a scaling of this parameter.

Global BHG and $\widehat{\text{BHG}}$. Here $\sigma_0 = \tau$ and for the BHG brane solutions, $\dot\phi = -\frac{2l}{\omega\lambda}$ while $\dot\xi_1 = -\frac{2\omega}{l\lambda}$. Supersymmetry dictates $\rho = 0$, i.e. the branes sit at the 'centre' of the global AdS_2. The BPS condition reads

$$H_G = \frac{2l}{\omega\lambda}|\Pi_\phi| + \frac{2\omega}{l\lambda}|\Pi_{\xi_1}| \,, \tag{2.15}$$

which is a function of α. Here[2] and in what follows we have defined $\Pi_x = P_x - A_x$ where A_x is obtained from the WZ term by writing it as $\dot x A_x$. Later, we will denote the energy density by $\mathcal{H}$ and the momentum densities corresponding to Π_x's by $\mathcal{P}_x$, so that

$$\Pi_x = \int_{\text{D3}} \mathcal{P}_x \, d\sigma_1 d\sigma_2 d\sigma_3 \,. \tag{2.16}$$

We find the non-zero momenta

$$P_\phi = -T_3 \frac{2\pi^2}{3} l^4 \cos^2\alpha \,, \quad P_\psi = P_\phi \cos\theta \,, \quad P_{\xi_1} = -T_3 2\pi^2 l^4 \cos^2\alpha \,, \tag{2.17}$$

with T_3 being the D3 brane tension. In the case of the $\widehat{\text{BHG}}$s, $\dot\xi_i = -\frac{2\omega}{l\lambda}$ and the BPS condition reads

$$H_{\text{DG}} = \frac{2\omega}{l\lambda} (|\Pi_{\xi_1}| + |\Pi_{\xi_2}| + |\Pi_{\xi_3}|) \,. \tag{2.18}$$

In this case H_{DG} is a constant and the non-zero momenta are

$$P_{\xi_i} = -T_3 \pi^2 \omega^2 \left[\frac{4\omega^2}{a^2 b^2} - (\omega^2 + 2l^2)\right] \mu_i^2 \,. \tag{2.19}$$

The second piece proportional to $(\omega^2 + 2l^2)$ comes from the four-form potential and does not appear in the Π's. Furthermore, supersymmetry analysis dictates that there exists a

---

[2]In the case of a point particle coupled to a gauge field the Hamiltonian is given by $(p - A)^2/2m$ where $p = \partial L/\partial\dot x$. It is the combination of $(p - A)$ that ensures gauge invariance.

gauge choice where this term can be gauged away and hence $\Pi_{\xi_i} = P_{\xi_i}$ in this case. In both cases, the preserved supersymmetry satisfies $\epsilon_0^+ = \Gamma_{49}\epsilon_0^-$ for branes with $M\epsilon_0^\pm = \pm\epsilon_0^\pm$. The conserved spinor can be simplified to

$$\epsilon = e^{\mp\frac{i\tau}{2b}}(1 \mp \Gamma_{49})\epsilon_0^+ \,, \qquad (2.20)$$

where the upper sign is for branes and the lower sign for anti-branes. The bilinear of this spinor leads to the BPS condition

$$H = \frac{2\omega}{l\lambda}(|\Pi_{\xi_1}| + |\Pi_{\xi_2}| + |\Pi_{\xi_3}|) + \frac{2l}{\omega\lambda}|\Pi_\phi| \,, \qquad (2.21)$$

where we identify $H = \partial_\tau$, $\Pi_{\xi_i} = \partial_{\xi_i}$ and $\Pi_\phi = \partial_\phi$.

As is now clear, none of the sets of branes without fluxes has all four quantum numbers non-zero.

The missing quantum number may be realised by electromagnetic or mechanical waves. The former involves turning on world-volume fluxes and latter deformations of the induced metric. If the missing quantum number is to be provided by waves, then (2.21) predicts that for $\widehat{\mathrm{BHG}}$ there should be a wave along ϕ direction with velocity $\frac{2l}{\omega\lambda}$ while for BHG there should a wave along $\xi_2 + \xi_3$ direction with velocity $\frac{2\omega}{l\lambda}$. We will see that this is precisely the case.

3. Counting giants[3]

The promotion of the BPS condition (2.21) to a quantum condition suggests that the resulting quantum state may contain both giant and dual giant parts. If there is a duality between the two, which has yet to be established in the black hole context, then it should be possible to describe the quantum states using dual giants or giants alone. In this section we quantise the $\widehat{\mathrm{BHG}}$ space of solutions in global coordinates described above and compare the result to the macroscopic entropy formula (1.1). If we counted the Poincaré $\widehat{\mathrm{BHG}}$s we would get a divergence since all values of r give the same energy.

The microstates of the black hole are conjectured to be a collection of giant and/or dual giant gravitons. These branes correspond to D3-dipoles and carry no net charge but they will still locally excite the five-form field. Hence when integrated over a small five-dimensional surface which encloses a portion of the wrapped brane, the result will be proportional to the number of D3-branes enclosed [27]. With this picture in mind, we will integrate components of F over various spatial coordinates and use

$$\int F = 16\pi G_{10} T_3\, n \,, \qquad (3.1)$$

with $n \in \mathbb{Z}$ in order to determine quantisation conditions. Using

$$G_{10} = \frac{\pi^4 l^8}{2N^2} \,, \quad T_3 = \frac{N}{2\pi^2 l^4} \,, \qquad (3.2)$$

---

[3]We thank N. Suryanarayana for collaboration in this section

we have

$$16\pi G_{10} T_3 = \frac{4\pi^3 l^4}{N}\,. \tag{3.3}$$

Here N is an integer obtained after integrating $F_{\alpha\beta\xi_1\xi_2\xi_3} = 4l^4 \cos^3\alpha \sin\alpha \sin\beta \cos\beta$. Integrating

$$F_{\theta\phi\psi\alpha\xi_1} = \frac{q}{16} \sin\theta \sin\alpha \cos\alpha \tag{3.4}$$

with $q = -2\omega^2(\omega^2 + 2l^2)$ which is proportional to the electric charge gives

$$\frac{N}{2l^4}\omega^2(\omega^2 + 2l^2) = n_1 = \frac{N|q|}{4l^4}\,, \tag{3.5}$$

while integrating

$$F_{\phi\alpha\beta\xi_1\xi_2} = -\omega^2 l^2 \cos^3\alpha \sin\alpha \sin\beta \cos\beta\,, \tag{3.6}$$

gives

$$\frac{N\omega^2}{2l^2} = n_2\,. \tag{3.7}$$

These together imply that $\frac{N\omega^4}{2l^4} = \frac{2n_2^2}{N}$ is also an integer. Note that n_1 and n_2 are not independent but satisfy $n_1 = 2n_2 + 2n_2^2/N$. In terms of n_2, the entropy can be rewritten as

$$S_{\text{BH}} = \pi \left(\frac{N\omega^2}{l^2}\right)^{3/2} \sqrt{N + \frac{3N\omega^2}{4l^2}} = 2\pi\sqrt{2n_2^3\left(N + \frac{3}{2}n_2\right)}\,. \tag{3.8}$$

Here we have used $V_5 = \pi^3 l^5$ and $G_{10} = V_5 G_5$. We want to compare this entropy to a microscopic state counting using the microstates described in section 2. The gauge-invariant Hamiltonian for a single dual giant is given by (2.18). Furthermore, by solving the κ-symmetry constraint as in [23, 26, 28, 29], one can show that supersymmetry dictates the following constraints

$$\rho = 0\,, \quad P_\rho = 0\,, \quad \Pi_\alpha = 0\,, \quad \Pi_\beta = 0\,, \quad \Pi_{\xi_i} - c\mu_i^2 = 0\,, \tag{3.9}$$

where

$$\mu_1^2 + \mu_2^2 + \mu_3^2 = 1\,, \tag{3.10}$$

which can be treated as an additional constraint. Here

$$c = -\frac{V_3}{8\pi^2}\frac{N\omega^4}{l^4}\left(1 + \frac{3\omega^2}{4l^2}\right) = -\frac{V_3}{8\pi^2}\frac{4n_2^2}{N}\left(1 + \frac{3n_2}{2N}\right)\,. \tag{3.11}$$

Here V_3 is the volume factor obtained after integrating over the spatial world-volume coordinates. The integration over the full range gives $16\pi^2$. We will leave it undetermined for now. Following Dirac's procedure for 2nd class constraints, one can simply drop ρ, P_ρ from the phase space. After quantisation, the remaining constraints can be thought to be imposed on the Hilbert space satisfying the gauge-invariant bracket $[\Pi_a, x^a] = -i$. Demanding this canonical commutation relation gives us

$$[c\mu_i^2, \xi_j] = -i\delta_{ij}\,. \tag{3.12}$$

Defining the classical variables $\zeta_i = \sqrt{|c|}\mu_i e^{i\xi_i}$ and promoting them to quantum operators gives us the oscillator brackets[4]

$$[\zeta_i, \zeta_j^\dagger] = \delta_{ij}\,. \tag{3.13}$$

This leads to writing the quantum Hamiltonian as

$$H = \frac{2\omega}{l\lambda}(\zeta_i^\dagger \zeta_i) = \frac{2\omega}{l\lambda}(N_1 + N_2 + N_3)\,. \tag{3.14}$$

Now imposing the restriction $\mu_1^2 + \mu_2^2 + \mu_3^2 = 1$ we see that the quantum states created by these oscillators are

$$|N_1, N_2, N_3\rangle = \prod_{i=1}^{3} \frac{(\zeta_i^\dagger)^{N_i}}{\sqrt{N_i!}}|\,\text{vac}\rangle \tag{3.15}$$

with occupation numbers satisfying

$$N_1 + N_2 + N_3 = |c|\,. \tag{3.16}$$

Thus we have constructed the Hilbert space of a constrained three-dimensional harmonic oscillator.

Instead of directly imposing the quantum commutator brackets we can also proceed by applying Dirac's procedure to deal with second-class constraints [31, 30, 26, 28]. Imposing (3.10) on the classical phase space implies the relation

$$\Pi_{\xi_1} + \Pi_{\xi_2} + \Pi_{\xi_3} = c\,,$$

which is a first-class constraint. Thus we can take

$$\Pi_\alpha = 0\,, \quad \Pi_\beta = 0\,, \quad \Pi_{\xi_2} = c\mu_2^2\,, \quad \Pi_{\xi_3} = c\mu_3^2 \tag{3.17}$$

as a system of second-class constraints. We define the Poisson brackets as $\{f, g\}_{\text{PB}} = \frac{\partial f}{\partial \Pi}\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial \Pi}$, which is the classical equivalent to the quantum condition $[\Pi_a, x^a] = -i$. This procedure can be justified by realizing that there exists a gauge for the four form $C^{(4)}$ and thus for the effective gauge potential A, in which the term giving rise to the q-piece in the momentum constraint drops out. Following this procedure, we get the following commutator brackets

$$[c\mu_p^2, \xi_q] = -i\delta_{pq}\,, \quad p, q = 2, 3\,. \tag{3.18}$$

With these we can define two oscillators $\zeta_2 = \sqrt{|c|}\mu_2 e^{i\xi_2}, \zeta_3 = \sqrt{|c|}\mu_3 e^{i\xi_3}$ which satisfy the algebra of two commuting simple harmonic oscillators. This then yields

$$|\Pi_1| = |c| - N_2 - N_3\,, \tag{3.19}$$

as before. Thus we again end up with the Hilbert space of a constrained three-dimensional harmonic oscillator, whose state counting is a three-coloured partitioning problem.

Integrating over a five-dimensional surface transverse to the dual-giant world volume will give us the total number of dual giants allowed in the geometry. The transverse

---

[4]The role of creation and annihilation operators gets interchanged when considering anti-branes.

coordinates are α, β, ξ_i and this leads to the maximum number of dual giants to be N. When we consider M multiple dual giant probes, we need to satisfy [26, 28, 29]

$$\sum_i^M \left(N_1^{(i)} + N_2^{(i)} + N_3^{(i)} \right) = M|c|. \tag{3.20}$$

In terms of the integer n_2 and N the right-hand side can be rewritten as

$$\frac{V_3}{8\pi^2} \frac{4Mn_2^2}{N^2} \left(N + \frac{3}{2}n_2 \right). \tag{3.21}$$

We need the three-coloured partition of this in the limit $N \gg M \gg 1$ which will give the entropy

$$S_{\text{probes}}^{\widehat{\text{BHG}}} = 2\pi \sqrt{\frac{V_3}{8\pi^2} \frac{2Mn_2^2}{N^2} \left(N + \frac{3}{2}n_2 \right)}. \tag{3.22}$$

Note that for this argument to make sense we need to make sure that the integer we are partitioning is much less than M as this is the upper limit on the sum. This leads to the condition $\omega \ll l$. When $M = N$ and with $V_3 = 16\pi^2$, we can associate this factor with the Landau degeneracy of BHG. For giants

$$\Pi_\phi = P_\phi - A_\phi = P_\phi + T_3 \frac{\pi^2}{2} \omega^2 l^2 \cos^4 \alpha,$$
$$\Pi_{\xi_1} = P_{\xi_1} - A_{\xi_1} = P_{\xi_1} - 2T_3 \pi^2 l^4 \cos^4 \alpha. \tag{3.23}$$

Thus the maximum integral quantum number associated with the state annihilated by P_ϕ is $n_2/2$ and that with P_{ξ_1} is N. There is an additional factor of 2 corresponding to the additional giant solutions [23] found at $\theta = 0, \pi$ which carry the same quantum numbers. In total we have a degeneracy factor of Nn_2. It remains to be seen if this is merely a coincidence. One possibility is that this degeneracy is to be perceived as the ground state degeneracy for each dual giant and hence $3Nn_2$ colours rather than 3. Putting together all the ingredients above, we get $S_{\text{probes}} = 2S_{\text{BH}}$.

Eventually we would like a more rigorous justification for this missing degeneracy we observed above. The next obvious question to ask is: What happens when one switches on world-volume electromagnetic flux? Since this is known to provide angular momentum, it is natural to suspect that the missing quantum number, in this case associated with Π_ϕ may arise from the electromagnetic field. Let us suggest the following way of counting motivated by [14] which leads to the same relation between S_{probes} and S_{BH} as in this section.

Adding a fourth quantum number. When we have Π_ϕ turned on, either by electromagnetic waves (as shown in section 4) or otherwise, the BPS relation suggests

$$\Pi_\phi + \frac{\omega^2}{l^2}(\Pi_{\xi_1} + \Pi_{\xi_2} + \Pi_{\xi_3}) = P, \tag{3.24}$$

where P denotes the total momentum. Meanwhile, we have from the probe analysis

$$\Pi_{\xi_1} + \Pi_{\xi_2} + \Pi_{\xi_3} = n. \tag{3.25}$$

This partitioning of n into three integer-valued momenta can be accomplished in $n^2/2$ ways when n is large. This can be achieved by taking $\omega \ll l$, but $N \gg 1$. As we will show in later sections Π_ϕ can be constructed out of two integers and hence keeping P fixed can be realized in $P - \frac{\omega^2}{l^2}n$ ways. Thus the total number of ways of satisfying the above conditions is given by

$$\frac{n^2}{2}\left(P - \frac{\omega^2}{l^2}n\right),\tag{3.26}$$

which is maximised w.r.t n for $\Pi_\phi = \frac{\omega^2}{2l^2}n$ which can be small compared with n and hence can be thought of as arising from small fluctuations. Now we anticipate that this momentum is going to be carried by open strings which are MN in number since there are M dual giant probes and N dual giants making the black hole. The bosonic moduli corresponding to α, β and fermionic moduli corresponding to the 2 preserved supersymmetries will contribute a factor of 3. The microscopic entropy arising from the partitioning of Π_ϕ is given by

$$S_{\text{probes}} = 2\pi\sqrt{\frac{3\Pi_\phi MN}{6}} = 4\pi\frac{M}{N}\sqrt{2n_2^3\left(N + \frac{3}{2}n_2\right)} = 2\frac{M}{N}S_{\text{BH}},\tag{3.27}$$

when[5] $n = M|c|$. Let us now explain why this relation is expected.

What does S_{probes} count? Let us observe that S_{BH} scales in terms of the number of dual giants N like $S_{\text{BH}} = f(\omega, l) N^2$. In the analysis leading to (3.27) we computed the entropy associated with inserting M probe branes into the near-horizon geometry of the black hole. When we insert M probes in the black hole geometry, these will form a new bound state with a higher entropy proportional to $(M + N)^2$. The open string degrees of freedom associated with this new bound state are MN in number. The M^2 and N^2 open strings ending on the same type of branes take into account the degrees of freedom associated with separating the objects. We are associating the degeneracy of the probes with the number of ways that the MN open strings can carry Π_ϕ. Then our computation should correspond to the difference in the entropy of the new bound state made of $M + N$ branes and the entropy when the probe and the black hole are far apart. This is given by

$$S_{\text{probes}} = f(\omega, l)\left[(N + M)^2 - N^2 - M^2\right] = 2NMf(\omega, l) = 2\frac{M}{N}S_{\text{BH}}.\tag{3.28}$$

Taking $N = M$, we arrive at the conclusion that $S_{\text{probes}} = 2S_{\text{BH}}$, the result that emerged from two independent computations above. We emphasise that our identification of dual giants gravitons and black hole microstates is conjectural, but we take the above results as encouraging evidence for such a connection. Eventually, it will be important to understand why dual giants, which are objects expanding in AdS are a valid microscopic description of a black hole.

We must also remind the reader that we have not demonstrated the counting by quantisation of the phase space of the BPS waves directly which we will leave as an open

---

[5]There may be an overall $O(1)$ factor having to do with the subtlety in counting independent open string states stretched between giants (see [32]) present.

problem. However, the existence of waves carrying the right velocity which preserve the same supersymmetry as the non-fluxed solution makes it very plausible that the above argument is at least on the right track.

To provide more evidence, we now need to demonstrate the existence of BPS modes carrying Π_ϕ which we turn to in the next section.

4. Supersymmetry and world volume fluxes

In this section we want to investigate the possibility of preserving some fraction of supersymmetry for D3-branes with non-trivial world-volume gauge field configurations.[6] These are governed by an action of the form

$$L = -T_3 \int \sqrt{-\det(h+F)} \, \mathrm{d}^4\sigma \mp T_3 \int C^{(4)} \,, \tag{4.1}$$

where in accordance with [23], the upper sign stands for a brane and the lower sign for an anti-brane and $C^{(4)}$ is the pull back of the space-time four form potential. We shall investigate the question of supersymmetry from the point of view of world-volume κ-symmetry transformations. In the presence of world-volume flux, the supersymmetry condition for a D3-brane is [34, 35]

$$\Gamma\epsilon = \epsilon \,, \tag{4.2}$$

with the general κ-symmetry projector

$$\Gamma = \frac{\epsilon^{ijkl}}{\sqrt{-\det(h+F)}} \left(\frac{1}{4!}\gamma_{ijkl}I - \frac{1}{4}F_{ij}\gamma_{kl}J + \frac{1}{8}F_{ij}F_{kl}I \right) \,, \tag{4.3}$$

where

$$I\epsilon = -i\epsilon \,, \tag{4.4}$$

$$J\epsilon = i\epsilon^* \,. \tag{4.5}$$

For an anti-brane the right hand side of (4.2) has the opposite sign. Note that this simplifies to the condition (6.2) of [23] in the absence of world-volume fluxes, as required. Since we want to preserve the same supersymmetries as in the $F = 0$ case in [23], we must demand that

$$\epsilon^{ijkl}F_{ij}\gamma_{kl}\epsilon^* = 0, \qquad \epsilon^{ijkl}F_{ij}F_{kl} = 0 \,. \tag{4.6}$$

Define the world-volume field strength tensor as

$$F = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \,. \tag{4.7}$$

---

[6]For earlier work on world-volume fields in the context of giant gravitons see [33].

Since we are in four space-time dimensions we can split F into electric and magnetic fields. Then the second of the conditions (4.6), *i.e.* $F \wedge F = 0$, implies that $\mathbf{E}$ and $\mathbf{B}$ are orthogonal to each other. The first condition above implies

$$(E_1 \gamma_{23} - E_2 \gamma_{13} + E_3 \gamma_{12} + B_3 \gamma_{03} + B_2 \gamma_{02} + B_1 \gamma_{01}) \epsilon^* = 0 \,. \qquad (4.8)$$

After solving the supersymmetry constraint, it is still necessary to check the equations of motion. The embedding coordinates' equations of motion follow from varying the action (4.1). We choose to work in static gauge, aligning the four world-volume coordinates with certain space-time coordinates. Which set of space-time coordinates we choose will vary from case to case. The gauge field equations of motion are compactly given by the expression [36]

$$\partial_i \left(\sqrt{-\det(h+F)} \left\{ (h+F)^{-1} - (h-F)^{-1} \right\}^{ij} \right) = 0 \,. \qquad (4.9)$$

Finally, the Bianchi identities of the world-volume gauge fields, $dF = 0$ must also be satisfied.

As a warmup to the near-horizon geometry, but also because the result is interesting in its own right, we shall now analyse dual giant gravitons in $\mathrm{AdS}_5 \times \mathrm{S}^5$ with world-volume fluxes. In [25], giant-graviton configurations in $\mathrm{AdS}_5 \times \mathrm{S}^5$ were constructed following the method of Mikhailov [37]. There is was found that it is possible to excite electric and magnetic fields on the brane without breaking any further supersymmteries. The gauge fields obey wave equations and contribute a momentum to the BPS relation via their Poynting vector. We realise this scenario on dual giant gravitons and find that turning on fluxes on dual $\frac{1}{8}$-BPS giant gravitons breaks the supersymmetry further to $\frac{1}{16}$, at least for the type of configuration we study.

4.1 Fluxes on $\frac{1}{8}$ BPS dual giants in $\mathbf{AdS}_5 \times \mathbf{S}^5$

We will closely follow [26] and the reader is referred to it for more details. The $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric is

$$ds^2 = -V dt^2 + \frac{1}{V} dr^2 + \sum_{i=1}^3 \frac{r^2}{4} \left(\sigma_i^L \right)^2 + l^2 \left(d\alpha^2 + \cos^2 \alpha \, d\beta^2 + \sum_{i=1}^3 \mu_i^2 d\xi_i^2 \right) \,, \qquad (4.10)$$

where $V = 1 + \frac{r^2}{l^2}$, $\mu_1 = \sin \alpha$, and $\{\mu_i\}$ and $\{\sigma_i^L\}$ have the same meaning as in section 2. After a coordinate transformation we can write the 3-sphere metric in the alternative form

$$r^2 (d\theta^2 + \cos \theta^2 d\phi_1^2 + \sin \theta^2 d\phi_2^2) \,. \qquad (4.11)$$

The world-volume coordinates are labelled by σ_i with $i = 0, 1, 2, 3$. We choose static gauge such that $t = \sigma_0, \theta = \sigma_1, \phi = \sigma_2, \psi = \sigma_3$. The world-volume gamma matrices for AdS_5 dual giants in the coordinates of (4.10) are

$$\gamma_0 = V^{1/2} \Gamma_0 + \sum \mu_i \Gamma_{6+i} \,, \qquad \gamma_1 = \frac{r}{2} (\sin \phi \Gamma_2 + \cos \phi \Gamma_3) \,,$$
$$\gamma_2 = \frac{r}{2} \Gamma_4 \,, \qquad \gamma_3 = \cos \theta \gamma_2 - \frac{r}{2} \sin \theta (\sin \phi \Gamma_3 - \cos \phi \Gamma_2) \,. \qquad (4.12)$$

In terms of the coordinates of (4.11), *i.e.* $\{t, r, \theta, \phi_1, \phi_2\}$ these gamma matrices are

$$\gamma_0 = V^{1/2}\Gamma_0 + \sum \mu_i \Gamma_{6+i}, \qquad\qquad \gamma_1 = r\Gamma_2,$$

$$\gamma_2 = r\cos\sigma_1 \Gamma_3, \qquad\qquad\qquad \gamma_3 = r\sin\sigma_1 \Gamma_4. \qquad (4.13)$$

In these coordinates the induced metric is diagonal. $\frac{1}{8}$-BPS dual giants without gauge fields satisfy

$$\gamma_0 \gamma_1 \gamma_2 \gamma_3 \epsilon = -i\sqrt{-\det h}\,\epsilon, \qquad (4.14)$$

where h is the induced metric on the world-volume of the brane. The spinor ϵ is subject to the projection conditions

$$\Gamma_{09}\epsilon = \epsilon, \quad \Gamma_{68}\epsilon = i\epsilon, \quad \Gamma_{57} = i\epsilon. \qquad (4.15)$$

We now wish to preserve a fraction of supersymmetry with non-trivial gauge fields. In order to solve (4.2) with Γ given by (4.3), we need to satisfy

$$\epsilon^{ijkl}F_{ij}\gamma_{kl}\epsilon^* = 0. \qquad (4.16)$$

In terms of (4.13) this condition becomes

$$\left[\left(E_1 - i\frac{\sqrt{-h}}{h_{22}h_{33}}B_1\right)\gamma_{23} - \left(E_2 - i\frac{\sqrt{-h}}{h_{11}h_{33}}B_2\right)\gamma_{13} + \left(E_3 - i\frac{\sqrt{-h}}{h_{11}h_{22}}B_3\right)\gamma_{12}\right]\epsilon^* = 0. \qquad (4.17)$$

By equating real and imaginary part of this equation to zero individually it follows that without imposing any further projection conditions, we need to set $\mathbf{E} = \mathbf{B} = 0$. However, if we impose the additional projection $\Gamma_{23}\epsilon = i\epsilon$ in the basis (4.12), then gauge fields obeying

$$E_2 = 0, \quad B_3\cos\theta = -B_2, \quad E_1 = -\frac{2}{l}B_3, \quad E_3 = \frac{2}{l}B_1. \qquad (4.18)$$

solve the $\kappa-$symmetry condition including world-volume fluxes. Because of the extra projection condition, turning on the gauge field leads to breaking more supersymmetries. In the specific case above it leads to $\frac{1}{16}$-BPS states. As a consequence of supersymmetry, this configuration has $\mathbf{E}\cdot\mathbf{B} = 0$. The equations of motion for the embedding coordinates are solved by $\dot\xi_i = \frac{1}{l}, \dot\alpha = \dot\beta = \dot r = 0$, if the fields satisfy the equation

$$\partial_0 B_i - \frac{2}{l}\partial_2 B_i = 0, i = 1, 3. \qquad (4.19)$$

Hence we have waves moving with phase velocity $2/l$ in the ϕ direction. Finally, the gauge field equations of motion with the Bianchi identities give

$$\sin\theta\partial_\theta(E_1\sin\theta) - \cos\theta\partial_\phi E_3 + \partial_\psi E_3 = 0,$$
$$\partial_\theta E_3 - \partial_\psi E_1 + \partial_\phi E_1 \cos\theta = 0. \qquad (4.20)$$

As we will show in the next section these equations can be expressed compactly in terms of $\mathbb{C}P^1$ coordinates when the 3-sphere metric is explicitly written as a Hopf fibration. In

this picture the waves propagate along the fibre. With the constraints (4.18), we have the dramatic simplification

$$\sqrt{-\det h} = \sqrt{-\det(h + F)}\,.$$ (4.21)

These are analogous to the waves on $\frac{1}{8}$-BPS giant gravitons analysed by [25] with the difference that there the inclusion of waves did not break any further supersymmetries. The gauge field contributes canonical momenta

$$\mathcal{P}_{E_1} = \frac{\partial \mathcal{L}}{\partial E_1} = T_3 B_3 \sin\theta\,, \quad \mathcal{P}_{E_2} = \frac{\partial \mathcal{L}}{\partial E_2} = T_3 B_1 \cot\theta\,, \quad \mathcal{P}_{E_3} = \frac{\partial \mathcal{L}}{\partial E_3} = -T_3 B_1 \csc\theta\,.$$ (4.22)

The Hamiltonian density is given by

$$\mathcal{H} = \frac{1}{l}\left[|2\mathcal{P}_\phi| + \sum_{i=1}^{3} |\mathcal{P}_{\xi_i}|\right]\,,$$ (4.23)

where

$$\mathcal{P}_\phi = B_1 \mathcal{P}_{E_3} - B_3 \mathcal{P}_{E_1}\,.$$ (4.24)

and

$$\mathcal{P}_{\xi_i} = -T_3\left[\frac{l^2}{r^2 \sin\theta}(B_1^2 + B_3^2 \sin^2\theta)\mu_i^2 + r^2 l^2 \mu_i^2 \sin\theta\right]\,.$$ (4.25)

The angular momentum of the gauge field has introduced a new quantum number in addition to (J_1, J_2, J_3) leading to the four-tuple (S_1, J_1, J_2, J_3). When counting the degeneracy of such states, one focuses on states of fixed energy. Since the Hamiltonian is r dependent, there is a certain energy for each r, P_ϕ. The total number of ways of choosing r, P_ϕ to achieve this energy after quantisation corresponds to the degeneracy of these solutions. If $P_\phi = 0$ then each value of r corresponds to a different energy and the degeneracy is unity. With $P_\phi \neq 0$ turned on, we get a larger degeneracy since now different choices for r, P_ϕ can give the same energy. It would be interesting to carry out the quantisation of the new phase space and count these objects. We will not attempt to do so in this paper. However, let us attempt to motivate how these asymptotic states could be used to account for the microscopic entropy. Firstly, we have

$$El = 2P_\phi + P_{\xi_1} + P_{\xi_2} + P_{\xi_3}\,,$$ (4.26)

with

$$P_{\xi_1} + P_{\xi_3} + P_{\xi_3} = n\,,$$ (4.27)

which can be realized in $n^2/2$ ways. It is natural to identify E with the mass of the black hole which is known to be

$$M = \frac{3\pi\omega^2}{4G_5}\left(1 + \frac{3\omega^2}{2l^2} + \frac{2\omega^4}{3l^4}\right)\,.$$ (4.28)

The total number of ways in which the above constraints can be satisfied is $(El - n)n^2/2$ ways which is maximised when $El = (3/2)n$. Comparing now the mass of the black hole with this, we have

$$n = \frac{\omega^2 N^2}{l^2}\left(1 + \frac{3\omega^2}{2l^2} + \frac{2\omega^4}{3l^4}\right)\,,$$ (4.29)

with $P_\phi = n/4$. Assuming now that this is carried by N^2 open strings with a central charge of 3 arising from α, β and 2 supersymmetries, we have the microscopic entropy given by

$$S_{\text{micro}} = 2\pi \sqrt{\frac{\omega^2 N^4}{4l^2} \left(1 + \frac{3\omega^2}{2l^2} + \frac{2\omega^4}{3l^4}\right)}, \qquad (4.30)$$

which agrees with S_{BH} to leading order when $\omega \ll l$ but differs at higher orders. It will be nice to derive the analogous formula by quantising the phase space of solutions rather than by this indirect way.

4.2 General solution

It will turn out that the differential equations obeyed by the BHG and $\widehat{\text{BHG}}$ configurations we are about to investigate can be transformed into an equivalent form both in Poincarè and global coordinates. Before we analyse particular instances of BHG and $\widehat{\text{BHG}}$ configurations, we present here the general solution to these equations. Let us introduce the complex variable

$$z = 2e^{i\psi} \tan \frac{\theta}{2}. \qquad (4.31)$$

Then

$$\partial_\theta = \frac{1}{\sqrt{z\bar{z}}} \left(1 + \frac{z\bar{z}}{4}\right) (z\partial_z + \bar{z}\partial_{\bar{z}}), \qquad (4.32)$$

and

$$\partial_\psi = iz\partial_z - i\bar{z}\partial_{\bar{z}}. \qquad (4.33)$$

For later convenience let us briefly describe the geometry of these coordinates, in terms of which the metric on a squashed three sphere of radius R reads

$$d\Omega_3^2 = \frac{R^2}{4} \left[\frac{16 dz d\bar{z}}{(4 + z\bar{z})^2} + q^2 (d\phi + \mathcal{A}(z, \bar{z}))^2\right], \qquad (4.34)$$

where $\mathcal{A}$ is a one form that lives purely in the $\mathbb{C}P^1$ base, parametrised by z and $\bar{z}$. We have

$$\mathcal{A} = \frac{1}{2iV} \left(z^{-1}dz - \bar{z}^{-1}d\bar{z}\right). \qquad (4.35)$$

Here

$$V = \frac{4 + z\bar{z}}{4 - z\bar{z}}. \qquad (4.36)$$

The squashing parameter q is unity for the round three sphere and is determined for the solutions, together with the radius R, in terms of the AdS length l and rotation parameter ω. The equations we want to solve take the form (4.20) and can be compactly written as

$$2Vz\partial_z G = -i\partial_\phi G, \qquad 2V\bar{z}\partial_{\bar{z}}\bar{G} = i\partial_\phi\bar{G}, \qquad (4.37)$$

where G is a complex field, in the AdS$_5$ case of the previous section, $G = E_3 + i\sin\theta E_1$. Evidently one is the complex conjugate of the other. We now obtain the general solution

of (4.37). We expand $G(z, \bar{z}; \phi)$ in eigenmodes of the ∂_ϕ operator, keeping in mind the 4π periodicity of ϕ:

$$G(z, \bar{z}; \phi) = \sum_{k=-\infty}^{\infty} G_k(z, \bar{z}) e^{-\frac{ik}{2}\phi} . \tag{4.38}$$

This leads to the equation

$$\partial_z \ln G_k(z, \bar{z}) = \frac{ik}{2} \mathcal{A}_z(z) \tag{4.39}$$

with solution

$$G_k = \bar{g}_k(\bar{z}) \exp \left[\frac{ik}{2} \int \mathcal{A}_z dz \right] . \tag{4.40}$$

Here $\bar{g}_k(\bar{z})$ is an arbitrary anti-holomorphic function, i.e. independent of z. Regularity at $\theta = 0, \pi$ dictates that it take the form

$$\bar{g}_k(\bar{z}) = \sum_{n=-k/2}^{k/2} a_{k,n} \bar{z}^n . \tag{4.41}$$

Here k is an integer, so that the allowed values for n are integers and half-odd integers. giving a degeneracy of $2k + 1$ for each k. Thus for a given ϕ momentum k we have a degeneracy of $2k + 1$ in the sense that there are $2k + 1$ "independent" coefficients that determine $\bar{G}$. The electromagnetic fields on the BHGs and $\widehat{\mathrm{BHG}}$s may be quantised by treating them as small fluctuations around the zero-field vacuum in a fashion analogous to [12]. Upon quantisation the expansion coefficients $a_{k,n}$ and $a^*_{k,n}$ become creation and annihilation operators, from which we may construct two additional number operators that correspond to the excitations of the complex field G.

The integral in (4.40) may be done explicitly yielding

$$G_k(z, \bar{z}) = \bar{g}_k(\bar{z}) \left[\frac{z\bar{z}}{(4 + z\bar{z})^2} \right]^{\frac{k}{4}} . \tag{4.42}$$

4.3 Black hole giants with fluxes

We now turn to specific examples of compact D3-brane configurations with non-trivial world-volume fluxes in the near-horizon geometry. The analysis in the sections below applies to the case of a brane.

4.3.1 Global $\widehat{\mathrm{BHG}}$

Consider a D3-brane with world-volume coordinates $\sigma = \{\tau, \theta, \phi, \psi\}$ in static gauge. Furthermore, we assume that the embedding coordinates $X^m(\sigma_0)$ depend on time only and obey

$$\dot{\psi} = 0, \quad \dot{\phi} = 0, \quad \dot{\xi}_i = -\frac{2\omega}{l\lambda} \quad \dot{\alpha} = 0 \quad \dot{\rho} = 0 \tag{4.43}$$

In the absence of flux, supersymmetry further dictates $\rho = 0$, a feature that carries to the fluxed solutions. It can be shown (see appendix for details of the computation) that (4.8) leads to the condition that the fields satisfy

$$E_2 = 0, \quad B_3 \cos\theta = -B_2 \quad E_1 = -\frac{2l}{\omega\lambda} B_3, \quad E_3 = \frac{2l}{\omega\lambda} B_1 . \tag{4.44}$$

With these constraints there occurs a significant simplification of the on-shell DBI action. We find that

$$\sqrt{-\det\left(h+F\right)}=\sqrt{-\det h}\,.$$

Furthermore, it is evident that the field configurations above satisfy $\mathbf{E}\cdot\mathbf{B}=0$. From these relations it follows that these solutions preserve the *same* supersymmetries as the un-fluxed case found in [23]. We now demonstrate that the above configurations are indeed solutions to the equations of motion subject to certain further equations that can be solved in general. The equations of motion for the embedding coordinates can be shown to be satisfied if the two independent components (we choose to solve the constraints for E_1 and E_3) of the field strength satisfy

$$\partial_0 E_i - \frac{2l}{\omega\lambda}\partial_2 E_i = 0\,,\quad i=1,3\,. \tag{4.45}$$

From the supersymmetry constraints above it follows that the B_i satisfy a set of analogous equations. In addition to these we must also make sure that the gauge field on the brane obeys the Bianchi identities

$$\partial_2 E_1 - \frac{\omega\lambda}{2l}\partial_0 E_1 = 0\,,\qquad\qquad \partial_2 E_3 - \frac{\omega\lambda}{2l}\partial_0 E_3 = 0\,,$$
$$\partial_3 E_1 - \partial_1 E_3 - \frac{\omega\lambda}{2l}\partial_0 E_1\cos\theta = 0\,,\qquad \partial_3 E_1 - \partial_1 E_3 - \partial_2 E_1\cos\theta = 0\,. \tag{4.46}$$

and equations of motion (4.9). The first two are identical to the coordinate equations of motion. Combining the non-trivial information from the Bianchi identities with the gauge-field equations of motion leaves us with solving the system of partial differential equations

$$\sin\theta\partial_1(E_1\sin\theta) - \cos\theta\partial_2 E_3 + \partial_3 E_3 = 0\,,$$
$$\partial_1 E_3 - \partial_3 E_1 + \partial_1 E_1\cos\theta = 0\,. \tag{4.47}$$

Note that these are precisely the same as (4.20). The time dependence is given by (4.45), so that $E_i(\tau,\phi,\psi)=E_i(\sigma^+,\psi)$, where we have defined the light-cone variable $\sigma^+ = \frac{2l}{\omega\lambda}\tau + \phi$.

   Thus, physically, these solutions correspond to waves travelling with a phase-velocity that is exactly in accordance with the general BPS relation (2.21).

   The gauge field gives rise to the conjugate momentum densities

$$\mathcal{P}_{E_1} = \frac{\partial\mathcal{L}}{\partial E_1} = -T_3 B_3\sin\theta\,,$$
$$\mathcal{P}_{E_2} = \frac{\partial\mathcal{L}}{\partial E_2} = -T_3 B_1\cot\theta\,,$$
$$\mathcal{P}_{E_3} = \frac{\partial\mathcal{L}}{\partial E_3} = T_3 B_1\csc\theta\,. \tag{4.48}$$

The Hamiltonian density is

$$\mathcal{H} = \frac{2l}{\omega\lambda}|\mathcal{P}_\phi| + \frac{2\omega}{l\lambda}\sum_{i=1}^{3}|\mathcal{P}_{\xi_i}|, \qquad (4.49)$$

where $\mathcal{P}_{\xi_i}$ denote unintegrated Π_{ξ_i} with

$$\mathcal{P}_\phi = B_1\mathcal{P}_{E_3} - B_3\mathcal{P}_{E_1} \qquad (4.50)$$

and

$$\mathcal{P}_{\xi_i} = -T_3\left(\frac{\omega}{4l}\right)^2\mu_i^2\csc\theta\left[12\lambda^2|G|^2 + \omega^2(4l^2 + 3\omega^2)\sin^2\theta\right], \qquad (4.51)$$

We have defined the quantity

$$G = E_3 + i\sin\theta E_1. \qquad (4.52)$$

Equation (4.49) reproduces the BPS condition (2.21) with all four charges. Notice that three of the charges are realized as 'orbital' angular momenta of the classical brane motion, whereas one is realized in terms of angular momentum carried by the gauge field on the brane. Rewriting (4.47) in terms of the new complex variables and taking linear combinations leads precisely to equations (4.37), whose solutions were obtained above.

4.3.2 Poincaré $\widehat{\text{BHG}}$

We shall now work in the coordinate system (2.3). Let us consider a $D3$-brane with world volume coordinates $\sigma = \{t, \theta, \phi, \psi\}$, where t is AdS$_2$ Poincaré time as defined in [23]. We assume static gauge and in addition that the remaining embedding coordinates are functions of σ_0 only. They satisfy

$$\dot{\xi}_i = \dot{\alpha} = \dot{\beta} = 0 \qquad (4.53)$$

From the analysis in [23] it follows that these satisfy $\gamma_0\epsilon^* = 0$. Using this, we find that (4.8) implies

$$E_i = 0, \qquad B_2 + B_3\cos\theta = 0 \qquad (4.54)$$

with B_1 unconstrained by supersymmetry. Here, the equations of motion and Bianchi identities reduce to the equations

$$\partial_1 B_1 - \cos\theta\partial_2 B_3 + \partial_3 B_3 = 0,$$
$$\sin\theta(\partial_1(\sin\theta B_3)) - \partial_3 B_1 + \cos\theta\partial_2 B_1 = 0, \qquad (4.55)$$

where all fields are time-independent as a result of the remaining Bianchi identities. Notice that the gauge field configuration on this kind of brane is like a 'snapshot' of the propagating wave found on the dual giant in global AdS$_2$ coordinates above. Defining $G = B_1 + i\sin\theta B_3$ and taking linear combination again yields (4.37). The mechanical momentum densities are

$$\mathcal{P}_{\xi_i} = T_3\left(\frac{l}{4\omega}\right)^2\csc\theta\left[16|G|^2 + \omega^4\sin^2\theta\right]\mu_i^2, \qquad (4.56)$$

while the field gives rise to

$$\mathcal{P}_{E_1} = \frac{\partial \mathcal{L}}{\partial E_1} = T_3 B_3 \sin\theta \,, \quad \mathcal{P}_{E_2} = \frac{\partial \mathcal{L}}{\partial E_2} = T_3 B_1 \cot\theta \,, \quad \mathcal{P}_{E_3} = \frac{\partial \mathcal{L}}{\partial E_3} = -T_3 B_1 \csc\theta \,.$$
(4.57)

As their un-fluxed counterparts, these configurations satisfy the BPS relation $\mathcal{H} = 0$. Let us now turn to giant-like configurations, i.e. configuration that wrap a submanifold in the S^5 part of the geometry.

4.3.3 Global BHG

Let us consider a giant-like configuration with world-volume coordinates $\sigma = \{\tau, \beta, \xi_2, \xi_3\}$. We want to put a non-trivial gauge field configuration on the solution in [23] with

$$\dot{\psi} = 0, \quad \dot{\phi} = -\frac{2l}{\omega\lambda}, \quad \dot{\xi}_1 = -\frac{2\omega}{l\lambda} \quad \dot{\alpha} = 0 \quad \dot{\rho} = 0$$
(4.58)

The supersymmetry constraints are solved by the relations

$$E_2 = -E_3 = -\frac{2\omega}{l\lambda} B_1 \,, \quad B_3 = -B_2 \tan^2\beta \,, \quad E_1 \cos^2\beta = \frac{2\omega}{l\lambda} B_2 \,.$$
(4.59)

The fact that $F \wedge F$ vanishes is again telling us that the electric and magnetic fields are perpendicular to one another. With these solutions, we see that again

$$\sqrt{-\det(h + F)} = \sqrt{-\det(h)} \,,$$

so that the same linear combination of supercharges is preserved with flux, as without flux. Thus, indeed we have an EM-wave running in the directions ξ_2 and ξ_3. The Bianchi identities and coordinate equations of motion determine the time dependence of the waves to be

$$\partial_3 E_i + \partial_2 E_i = \frac{l\lambda}{2\omega} \partial_0 E_i \,, \quad i = 2, 3 \,.$$
(4.60)

The phase velocity of the waves, $\frac{2\omega}{l\lambda}$, is again exactly as expected from the BPS condition.

The gauge field equations of motion (4.9) together with the Bianchi identities on the configuration under consideration here lead to the system of equations

$$\sin 2\beta \partial_1 (\sin 2\beta E_1) - 2\partial_3 E_2 + 2\partial_2 E_2 + \frac{l\lambda}{2\omega} \cos 2\beta \partial_0 E_2 = 0 \,,$$
$$\partial_2 E_1 - \partial_3 E_1 - 2\partial_1 E_2 + \frac{l\lambda}{\omega} \cos 2\beta \partial_0 E_1 = 0 \,.$$
(4.61)

Upon identifying $2\beta \to \theta$, $\xi_2 - \xi_3 \to \psi$ and $\xi_2 + \xi_3 \to \phi$ we can recast this computation into the standard form above. The quantity $G = 2E_2 + i \sin 2\beta E_1$ satisfies (4.37).

The mechanical momentum densities pick up contributions due to the field:

$$\mathcal{P}_\phi = -T_3 \frac{l^2}{24\omega^2} \frac{1}{\sin 2\beta \cos^2\alpha} \left(\lambda^2 |G|^2 + \omega^2 \cos^4\alpha \sin^2 2\beta (4l^2 + 3\omega^2 \cos^2\alpha) \right) \,,$$
(4.62)

$$\mathcal{P}_\psi = \cos\theta \mathcal{P}_\phi \,,$$
(4.63)

$$\mathcal{P}_{\xi_1} = -T_3 \frac{l^2}{8\omega^2} \frac{\tan^2\alpha}{\sin 2\beta} \left(\lambda^2 |G|^2 + 4l^2 \omega^2 \cos^4\alpha \sin^2 2\beta \right) \,.$$
(4.64)

Note that in the absence of G, there was an upper bound in the momenta. Since α runs between 0 and $\pi/2$ there is no such upper bound any more. This seems to hint at the interpretation of fluxes on giants as descendants [15]. However since AdS$_5$ dual giants with fluxes are not analogous to their S^5 counterparts, the corresponding interpretation of fluxes on dual giants as descendants is less clear. The gauge field degrees of freedom have conjugate momentum densities

$$\mathcal{P}_{E_1} = \frac{\partial \mathcal{L}}{\partial E_1} = T_3 B_2 \tan\beta \,, \quad \mathcal{P}_{E_2} = \frac{\partial \mathcal{L}}{\partial E_2} = -T_3 B_1 \cot\beta \,, \quad \mathcal{P}_{E_3} = \frac{\partial \mathcal{L}}{\partial E_3} = T_3 B_1 \tan\beta \,.$$

(4.65)

The Hamiltonian density gives rise to the BPS relation

$$\mathcal{H} = \frac{2l}{\omega\lambda}|\mathcal{P}_\phi| + \frac{2\omega}{l\lambda}\sum_{i=1}^{3}|\mathcal{P}_{\xi_i}| \,,$$

(4.66)

where

$$\mathcal{P}_{\xi_2} = B_3 \mathcal{P}_{E_1} - B_1 \mathcal{P}_{E_3} \,, \quad \mathcal{P}_{\xi_3} = B_1 \mathcal{P}_{E_2} - B_2 \mathcal{P}_{E_3} \,.$$

(4.67)

4.3.4 Poincaré BHG

Solving the supersymmetry constraints for $D3$ branes wrapping $\{t, \beta, \xi_2, \xi_3\}$ in AdS$_2$ Poincaré coordinates, where $\dot{\xi}_1 = \dot{\theta} = \dot{\phi} = \dot{\psi} = \dot{r} = 0$, results in the constraints

$$E_i = 0, \qquad B_3 + \tan^2\beta\, B_2 = 0$$

(4.68)

with B_1 unconstrained. Taking note of fact that $\gamma_0 \epsilon^* = 0$ (see [23]) simplifies the calculation. The DBI part of the action on this class of solutions again simplifies in the same way as above. The equations of motion for an anti-brane and associated Bianchi identities reduce to

$$\partial_2 B_2 - \cot^2\beta \partial_3 B_2 + \partial_1 B_1 = 0 \,,$$
$$\partial_1(\tan\beta B_2) + \partial_3 \tan\beta B_1 - \partial_2 \cot\beta B_1 = 0$$

(4.69)

with all magnetic field components time-independent. Defining the auxiliary variables

$$B_2 = \frac{l\lambda}{2\omega}\cos^2\beta G_1 \,, \qquad G_2 = -G_3 = -\frac{2\omega}{l\lambda}B_1 \,.$$

and identifying $2\beta \to \theta$, $\xi_2 - \xi_3 \to \psi$ and $\xi_2 + \xi_3 \to \phi$, after some algebra, transforms the equations into standard form (4.37) in terms of the complex field $G = 2G_3 + i\sin 2\beta G_1$. The mechanical momentum densities pick up contributions due to the field:

$$\mathcal{P}_\phi = T_3 \frac{1}{8\sin 2\beta \cos^2\alpha}(\lambda^2|G|^2 + l^2\omega^2\cos^4\alpha\sin^2 2\beta(4 - \cos^2\alpha)) \,,$$

(4.70)

$$\mathcal{P}_\psi = \cos\theta \mathcal{P}_\phi \,,$$

(4.71)

$$\mathcal{P}_{\xi_1} = T_3 \frac{l^2}{8\omega^2}\frac{\tan^2\alpha}{\sin 2\beta}(\lambda^2|G|^2 + 4l^2\omega^2\cos^4\alpha\sin^2 2\beta) \,.$$

(4.72)

As in the global case, the $\mathcal{P}_i$ do not have upper limits any more. The gauge field degrees of freedom have conjugate momentum densities

$$\mathcal{P}_{E_1} = \frac{\partial \mathcal{L}}{\partial E_1} = T_3 B_2 \tan \beta\,, \quad \mathcal{P}_{E_2} = \frac{\partial \mathcal{L}}{\partial E_2} = -T_3 B_1 \cot \beta\,, \quad \mathcal{P}_{E_3} = \frac{\partial \mathcal{L}}{\partial E_3} = T_3 B_1 \tan \beta\,. \tag{4.73}$$

The solutions satisfy the BPS relation $\mathcal{H} = 0$ in terms of their conjugate momenta as expected from the Killing spinor bilinear.

5. Discussion

In this paper we discussed microstates in the near-horizon geometry of a $\frac{1}{16}$-BPS AdS$_5$ black hole. We counted dual giant configurations in the probe approximation by quantising the phase space of solutions. The result missed the macroscopic entropy by a degeneracy factor. We argued that turning on an additional angular momentum quantum number, achieved in this paper by world-volume fluxes and dictated by the near-horizon supersymmetry, can potentially produce the correct statistical entropy.

We found a whole class of solutions preserving exactly the same supersymmetry as those without fluxes. These solutions are BPS electromagnetic waves and are entirely consistent with the supersymmetries of the near-horizon geometry. They have precisely the velocity predicted by supersymmetry and exist on the world volumes of both giants and dual giants. The resulting configurations carry all four quantum numbers dictated by supersymmetry. We also demonstrated that world-volume fluxes on $\frac{1}{8}$-BPS dual giants in AdS$_5 \times$ S^5 will generically lead to $\frac{1}{16}$-BPS configurations with an additional quantum number. It will be interesting to consider the partition functions of these states along the lines of [26].

The global $\widehat{\text{BHG}}$ configurations in this paper may be viewed as the caps of the microstates of the full black hole in the fuzzball [38] proposal.[7] It will be very interesting to consider the quantisation of the new space of near-horizon solutions and to see if the macroscopic entropy is reproduced. A simple minded maximisation argument was shown to lead to an exact match with the macroscopic entropy although we should emphasise that this cannot be construed as a satisfactory derivation as yet. What was crucial in this argument was the existence of the additional angular momentum quantum number, one source of which are the electromagnetic waves. There could be other sources such as vibrational modes which we have not ruled out. A related puzzle is that fluxes on $1/8 AdS_5 \times S^5$ giants are to be thought of as descendants [15] and as such would lead to double counting. If the same interpretation extends to $\frac{1}{16}$-BPS AdS$_5$ dual giants with fluxes, then these should be thought of as descendants of some chiral primary operators presumably corresponding to BPS vibrational modes [39]. However, since electromagnetic waves broke supersymmetry in the dual giant case, that this analogy holds is not clear to us. Although it is expected that BPS fluctuations or mechanical waves also should play a role in the counting of microstates, it is not implausible that the electromagnetic waves are just a dual description of these

---

[7]We thank N. Suryanarayana for suggesting this to us.

mechanical waves. Electromagnetic flux is related to open strings while the vibrational modes are related to the metric, so this would be similar in spirit to open-closed duality. Since it appears that finding solutions for mechanical waves is considerably harder, one could hope that counting the electromagnetic waves in a systematic way could reproduce the same result. Our analysis should be useful in studying these issues further.

Acknowledgments

A. Details of computations for dual giants

In this appendix we supply more detail on the supersymmetry analysis whose results were quoted in section 4.3.1. On the dual giant wrapping $\{\tau, \theta, \phi, \psi\}$ in AdS$_2$ global coordinates, we have the induced metric

$$
h = \begin{pmatrix}
-1 + J^2 l^2 & 0 & -\frac{J\omega^2}{4} & -\frac{1}{4}J\omega^2 \cos\sigma_1 \\
0 & \frac{\omega^2}{4} & 0 & 0 \\
-\frac{J\omega^2}{4} & 0 & \frac{\omega^2(l^2+\omega^2)}{4l^2} & \frac{\omega^2(l^2+\omega^2)}{4l^2}\cos\sigma_1 \\
-\frac{1}{4}J\omega^2 \cos\sigma_1 & 0 & \frac{\omega^2(l^2+\omega^2)}{4l^2}\cos\sigma_1 & \frac{\omega^2(2l^2+\omega^2+\omega^2\cos2\sigma_1)}{8l^2}
\end{pmatrix} \tag{A.1}
$$

where $J = \mp\frac{2\omega}{l\lambda}$ for a brane /anti brane. We wish to preserve the same linear combinations of supercharges as in the unfluxed case, discussed in [23]. The preserved Killing spinor satisfies the relation

$$
(h_{02} - \gamma_0\gamma_2)\epsilon = -b\epsilon \,,
$$

so that

$$
\gamma_0\epsilon^* = -\frac{b + h_{02}}{h_{22}}\gamma_2\epsilon^* \,. \tag{A.2}
$$

Using this we find that the vanishing of the $\epsilon^{ijkl}F_{ij}\gamma_{kl}$ term in the κ symmetry projector sets

$$
\left[E_1(-\gamma_3\gamma_2 + h_{23}) + E_2\gamma_1\gamma_3 + E_3\gamma_1\gamma_2 + B_3\left(\frac{b + h_{02}}{h_{22}}\gamma_3\gamma_2 + h_{03}\right) + \right. \tag{A.3}
$$

$$
\left. \left(b - \frac{J\omega^2}{2}\right)B_2 - B_1\frac{b + h_{02}}{h_{22}}\gamma_1\gamma_2 \right]\epsilon^* = 0 \,.
$$

Using the explicit form of the world-volume gamma matrices, we compute the various terms appearing above.

$$-\gamma_3\gamma_2 + h_{23} = \frac{\omega^2}{4l}\sin\theta(\sin\phi\Gamma_3 - \cos\phi\Gamma_2)\left(\frac{\omega}{2}\Gamma_9 + \frac{l}{ab}\Gamma_4\right),$$

$$\frac{b+h_{02}}{h_{22}}\gamma_3\gamma_2 + h_{03} = \left(b - \frac{J\omega^2}{2}\right)\cos\theta + \frac{b - \frac{J\omega^2}{4}}{h_{22}}\frac{\omega^2}{4l}\sin\theta(\sin\phi\Gamma_3 - \cos\phi\Gamma_2)\left(\frac{\omega}{2}\Gamma_9 + \frac{l}{ab}\Gamma_4\right),$$

$$\gamma_1\gamma_3 = \frac{\omega^2}{4l}(\cos\phi\Gamma_3 + \sin\phi\Gamma_2)\cos\theta\left(\frac{\omega}{2}\Gamma_9 + \frac{l}{ab}\Gamma_4\right) + \frac{\omega^2}{4}\sin\theta\Gamma_{23},$$

$$\gamma_1\gamma_2 = \frac{\omega^2}{4l}(\cos\phi\Gamma_3 + \sin\phi\Gamma_2)\left(\frac{\omega}{2}\Gamma_9 + \frac{l}{ab}\Gamma_4\right). \tag{A.4}$$

One now projects the resulting equations onto the subspace defined by the projection conditions for $\widehat{\text{BHG}}$ and equations independent generators of the Clifford algebra to zero individually. The constant term in equation (A.3) gives

$$\left(b - \frac{J\omega^2}{2}\right)(B_3\cos\theta + B_2) - i\frac{\omega^2}{4}\sin\theta E_2, \tag{A.5}$$

demanding this to be zero gives

$$E_2 = 0, \quad B_3\cos\theta = -B_2. \tag{A.6}$$

Further the coefficient (after using the projection condition $\Gamma_{23}\epsilon = i\epsilon$) of $e^{i\phi}\Gamma_2(\frac{\omega}{2}\Gamma_9 + \frac{l}{ab}\Gamma_4)$ gives

$$E_1 = -B_3\left(b - \frac{J\omega^2}{4}\right)\frac{1}{h_{22}} = -\frac{2l}{\omega\lambda}B_3, \tag{A.7}$$

$$E_3 = B_1\left(b - \frac{J\omega^2}{4}\right)\frac{1}{h_{22}} = \frac{2l}{\omega\lambda}B_1. \tag{A.8}$$

These are the relations used in section 4.3.1. The Poinaré computation goes ahead in much the same way, but is algebraically simpler.

B. Details of computations for giants

For a brane wrapping $\{\tau, \beta, \xi_2, \xi_3\}$ we have the induced metric

$$h_{\text{giant}} = \begin{pmatrix} \frac{2\omega^2}{\lambda^2}(\mu_1^2 - 1) & 0 & \frac{l\omega}{2\lambda}\mu_2^2 & \frac{l\omega}{2\lambda}\mu_3^2 \\ 0 & l^2(1 - \mu_1^2) & 0 & 0 \\ \frac{l\omega}{2\lambda}\mu_2^2 & 0 & l^2\mu_2^2 & 0 \\ \frac{l\omega}{2\lambda}\mu_3^2 & 0 & 0 & l^2\mu_3^2 \end{pmatrix}. \tag{B.1}$$

We compute

$$\gamma_1\gamma_2 = l^2\mu_2\left[\mu_3\Gamma_{68} - \mu_2\left(\sqrt{1 - \mu_1^2}\Gamma_{69} + \mu_1\Gamma_{67}\right)\right]$$

$$\gamma_1\gamma_3 = -l^2\mu_3\left[\mu_2\Gamma_{68} + \mu_3\left(\sqrt{1 - \mu_1^2}\Gamma_{69} + \mu_1\Gamma_{67}\right)\right]$$

$$\gamma_2\gamma_3 = -l^2\mu_2\mu_3\left[\sqrt{1 - \mu_1^2}\Gamma_{89} + \mu_1\Gamma_{87}\right]. \tag{B.2}$$

The supersymmetry constraint (4.8) now reads

$$\left[E_1\gamma_2\gamma_3 - E_2\gamma_1\gamma_3 + E_3\gamma_1\gamma_2 + B_3\left(-\gamma_3\gamma_0 + \frac{l\omega}{2\lambda}\mu_3^2\right) \right.$$
$$\left. + B_2\left(-\gamma_2\gamma_0 + \frac{l\omega}{2\lambda}\mu_2^2\right) - B_1\gamma_1\gamma_0 \right]\epsilon^* = 0. \qquad (B.3)$$

Since we want to preserve the same supersymmetries as without flux, we can use the fact that

$$\Gamma\epsilon^* = -\frac{1}{\sqrt{-\det h}}\left(\gamma_0\gamma_1\gamma_2\gamma_3 - h_{03}\gamma_1\gamma_2 + h_{02}\gamma_1\gamma_3\right)\epsilon^* = i\epsilon^* \qquad (B.4)$$

for the case of a brane. Plugging in the expressions for $\gamma_1\gamma_2$ and $\gamma_1\gamma_3$ from above, we arrive at the equation

$$\gamma_0\gamma_1\gamma_2\gamma_3\epsilon^* = -i\left(\sqrt{-h} - \frac{l^3\omega}{2\lambda}\mu_2\mu_3(1-\mu_1^2)\right)\epsilon^* \equiv -iA\epsilon^*. \qquad (B.5)$$

It is easy to compute

$$A = -\frac{l^2\lambda}{2\omega}\mu_2\mu_3 h_{00}. \qquad (B.6)$$

Now we may write the supersymmetry condition

$$\left[\left(E_1 - \frac{i}{A}h_{00}h_{11}B_1\right)\gamma_2\gamma_3 - \left(E_2 - \frac{i}{A}h_{00}h_{22}B_2\right)\gamma_1\gamma_3 \right. \qquad (B.7)$$
$$\left. + \left(E_3 - \frac{i}{A}h_{00}h_{33}B_3\right)\gamma_1\gamma_2 + \frac{l\omega}{2\lambda}\left(B_3\mu_3^2 + B_2\mu_2^2\right)\right]\epsilon^* = 0.$$

From this we may extract the coefficient equations of $\{\mathbb{1}, \Gamma_{69}, \Gamma_{67}\}$. Start with the coefficient of the unit matrix:

$$il^2\mu_2\mu_3(E_2 + E_3) + \frac{l^2 h_{00}}{A}\mu_2\mu_3\left(h_{22}B_2 + h_{33}B_3\right) + \frac{l\omega}{2\lambda}\left(B_3\mu_3^2 + B_2\mu_2^2\right) = 0. \qquad (B.8)$$

We must equate real and imaginary parts to zero individually and obtain

$$E_2 = -E_3 \qquad B_3\cos^2\beta = -B_2\sin^2\beta. \qquad (B.9)$$

Next, we turn to the coefficient of Γ_{67}:

$$-i\left(E_1 - \frac{i}{A}h_{00}h_{11}B_1\right)\mu_2\mu_3 + \mu_3^2\left(E_2 - \frac{i}{A}h_{00}h_{22}B_2\right) - \mu_2^2\left(E_3 - \frac{i}{A}h_{00}h_{33}B_3\right) = 0, \quad (B.10)$$

while the coefficient of Γ_{69} gives

$$-i\left(E_1 - \frac{i}{A}h_{00}h_{11}B_1\right)\mu_2\mu_3 + \mu_3^2\left(E_2 - \frac{i}{A}h_{00}h_{22}B_2\right) - \mu_2^2\left(E_3 - \frac{i}{A}h_{00}h_{33}B_3\right) = 0, \quad (B.11)$$

which are the same conditions. Equating the real and imaginary parts gives

$$-\frac{2\omega}{l\lambda}B_1 = E_2, \quad E_1 = \frac{2\omega}{l\lambda}(B_2 - B_3). \qquad (B.12)$$

A cross-check that these are correct is to compute

$$\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} = E_1 B_1 + E_2 B_2 + E_3 B_3 = 0 \,, \tag{B.13}$$

which is needed for susy to hold. Thus the most general solution is:

$$E_2 = -E_3 = -\frac{2\omega}{l\lambda} B_1 \,, \quad B_3 = -B_2 \tan^2\beta \,, \quad E_1 \cos^2\beta = \frac{2\omega}{l\lambda} B_2 \,. \tag{B.14}$$

This is the set of constraints used in section 4.3.3. Again, the Poincaré case proceeds analgously.

References

[1] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [`hep-th/9601029`].

[2] B. de Wit, *BPS black holes*, `arXiv:0704.1452`;
M. Guica and A. Strominger, *Cargese lectures on string theory with eight supercharges*, `arXiv:0704.3295`;
I. Bena and N.P. Warner, *Black holes, black rings and their microstates*, `hep-th/0701216`;
B. Pioline, *Lectures on on black holes, topological strings and quantum attractors*, *Class. Quant. Grav.* **23** (2006) 981 [`hep-th/0607227`];
P. Kraus, *Lectures on black holes and the AdS₃/CFT(2) correspondence*, `hep-th/0609074`;
S.D. Mathur, *The quantum structure of black holes*, *Class. and Quant. Grav.* **23** (2006) R115 [`hep-th/0510180`];
T. Mohaupt, *Black hole entropy, special geometry and strings*, *Fortschr. Phys.* **49** (2001) 3 [`hep-th/0007195`];
S.R. Das and S.D. Mathur, *The quantum physics of black holes: results from string theory*, *Ann. Rev. Nucl. Part. Sci.* **50** (2000) 153 [`gr-qc/0105063`];
J.M. Maldacena, *Black holes in string theory*, `hep-th/9607235`.

[3] J.B. Gutowski and H.S. Reall, *Supersymmetric AdS₅ black holes*, *JHEP* **02** (2004) 006 [`hep-th/0401042`].

[4] J.B. Gutowski and H.S. Reall, *General supersymmetric AdS₅ black holes*, *JHEP* **04** (2004) 048 [`hep-th/0401129`].

[5] Z.W. Chong, M. Cvetič, H. Lu and C.N. Pope, *Five-dimensional gauged supergravity black holes with independent rotation parameters*, *Phys. Rev.* **D 72** (2005) 041901 [`hep-th/0505112`].

[6] Z.W. Chong, M. Cvetič, H. Lu and C.N. Pope, *General non-extremal rotating black holes in minimal five-dimensional gauged supergravity*, *Phys. Rev. Lett.* **95** (2005) 161301 [`hep-th/0506029`].

[7] H.K. Kunduri, J. Lucietti and H.S. Reall, *Supersymmetric multi-charge AdS₅ black holes*, *JHEP* **04** (2006) 036 [`hep-th/0601156`].

[8] P. Figueras, C.A.R. Herdeiro and F.P. Correia, *On a class of 4D Kahler bases and AdS₅ supersymmetric black holes*, *JHEP* **11** (2006) 036 [`hep-th/0608201`].

[9] M.T. Grisaru, R.C. Myers and O. Tafjord, *SUSY and Goliath*, *JHEP* **08** (2000) 040 [`hep-th/0008015`].

[10] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the giant gravitons from Anti-de Sitter space*, *JHEP* **06** (2000) 008 [hep-th/0003075].

[11] J.P. Gauntlett, J.B. Gutowski and N.V. Suryanarayana, *A deformation of $AdS_5 \times S^5$*, *Class. and Quant. Grav.* **21** (2004) 5021 [hep-th/0406188].

[12] S. Kim and K.-M. Lee, *1/16-BPS black holes and giant gravitons in the $AdS_5 \times S^5$ space*, *JHEP* **12** (2006) 077 [hep-th/0607085].

[13] S. Benvenuti, B. Feng, A. Hanany and Y.-H. He, *Counting BPS operators in gauge theories: quivers, syzygies and plethystics*, hep-th/0608050;
B. Feng, A. Hanany and Y.-H. He, *Counting gauge invariants: the plethystic program*, *JHEP* **03** (2007) 090 [hep-th/0701063];
A. Hanany and C. Romelsberger, *Counting BPS operators in the chiral ring of $N = 2$ supersymmetric gauge theories or $N = 2$ brane surgery*, hep-th/0611346;
A. Butti, D. Forcella and A. Zaffaroni, *Counting BPS baryonic operators in CFTs with Sasaki-Einstein duals*, hep-th/0611229; *Baryonic generating functions*, hep-th/0701236.

[14] J. Kinney, J.M. Maldacena, S. Minwalla and S. Raju, *An index for 4 dimensional super conformal theories*, hep-th/0510251.

[15] I. Biswas, D. Gaiotto, S. Lahiri and S. Minwalla, *Supersymmetric states of $N = 4$ Yang-Mills from giant gravitons*, hep-th/0606087.

[16] P.J. Silva, *Thermodynamics at the BPS bound for black holes in AdS*, *JHEP* **10** (2006) 022 [hep-th/0607056].

[17] O.J.C. Dias and P.J. Silva, *Attractors and the quantum statistical relation for extreme (BPS or not) black holes*, arXiv:0704.1205.

[18] M. Berkooz, D. Reichmann and J. Simon, *A Fermi surface model for large supersymmetric AdS_5 black holes*, *JHEP* **01** (2007) 048 [hep-th/0604023].

[19] D. Gaiotto, A. Simons, A. Strominger and X. Yin, *D0-branes in black hole attractors*, hep-th/0412179.

[20] D. Gaiotto, A. Strominger and X. Yin, *Superconformal black hole quantum mechanics*, *JHEP* **11** (2005) 017 [hep-th/0412322].

[21] S. Fubini and E. Rabinovici, *Superconformal quantum mechanics*, *Nucl. Phys.* **B 245** (1984) 17.

[22] P. Claus et al., *Black holes and superconformal mechanics*, *Phys. Rev. Lett.* **81** (1998) 4553 [hep-th/9804177].

[23] A. Sinha, J. Sonner and N.V. Suryanarayana, *At the horizon of a supersymmetric AdS_5 black hole: isometries and half-BPS giants*, *JHEP* **01** (2007) 087 [hep-th/0610002].

[24] N.V. Suryanarayana and M.C. Wapler, *Charges from attractors*, arXiv:0704.0955.

[25] S. Kim and K.-M. Lee, *BPS electromagnetic waves on giant gravitons*, *JHEP* **10** (2005) 111 [hep-th/0502007].

[26] G. Mandal and N.V. Suryanarayana, *Counting 1/8-BPS dual-giants*, *JHEP* **03** (2007) 031 [hep-th/0606088].

[27] R.C. Myers and O. Tafjord, *Superstars and giant gravitons*, *JHEP* **11** (2001) 009 [hep-th/0109127].

[28] D. Martelli and J. Sparks, *Dual giant gravitons in Sasaki-Einstein backgrounds*, *Nucl. Phys.* **B 759** (2006) 292 [`hep-th/0608060`].

[29] A. Basu and G. Mandal, *Dual giant gravitons in $AdS_m \times Y^n$ (Sasaki-Einstein)*, `hep-th/0608093`.

[30] G. Mandal, *Fermions from half-BPS supergravity*, *JHEP* **08** (2005) 052 [`hep-th/0502104`].

[31] S.R. Das, A. Jevicki and S.D. Mathur, *Giant gravitons, BPS bounds and noncommutativity*, *Phys. Rev.* **D 63** (2001) 044001 [`hep-th/0008088`].

[32] V. Balasubramanian, D. Berenstein, B. Feng and M.-X. Huang, *D-branes in Yang-Mills theory and emergent gauge symmetry*, *JHEP* **03** (2005) 006 [`hep-th/0411205`].

[33] D. Sadri and M.M. Sheikh-Jabbari, *Giant hedge-hogs: spikes on giant gravitons*, *Nucl. Phys.* **B 687** (2004) 161 [`hep-th/0312155`];
S. Prokushkin and M.M. Sheikh-Jabbari, *Squashed giants: bound states of giant gravitons*, *JHEP* **07** (2004) 077 [`hep-th/0406053`].

[34] M. Cederwall, A. von Gussich, B.E.W. Nilsson and A. Westerberg, *The Dirichlet super-three-brane in ten-dimensional type-IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 163 [`hep-th/9610148`].

[35] E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [`hep-th/9611173`].

[36] M. Aganagic, C. Popescu and J.H. Schwarz, *Gauge-invariant and gauge-fixed D-brane actions*, *Nucl. Phys.* **B 495** (1997) 99 [`hep-th/9612080`].

[37] A. Mikhailov, *Giant gravitons from holomorphic surfaces*, *JHEP* **11** (2000) 027 [`hep-th/0010206`].

[38] S.D. Mathur, *The fuzzball proposal for black holes: an elementary review*, *Fortschr. Phys.* **53** (2005) 793 [`hep-th/0502050`].

[39] S.R. Das, A. Jevicki and S.D. Mathur, *Vibration modes of giant gravitons*, *Phys. Rev.* **D 63** (2001) 024013 [`hep-th/0009019`].